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Exp-function Method for Wick-type Stochastic Combined KdV-mKdV Equations

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Abstract: *Exp-function method is proposed to present soliton and periodic wave solutions for variable coefficients combined KdV- mKdV equation. By means of Hermite transform and white noise analysis, we consider the variable coefficients and Wick-type stochastic combined KdV-mKdV equations. As a result, we can construct new and more general formal solutions. These solutions include exact stochastic soliton and periodic wave solutions.*

Keywords: *combined KdV-mKdV equation, Exp-function method, Wick product, Hermite transform, White noise.*

Introduction

In this paper, we investigate the variable coefficients combined KdV-mKdV equation:

$$u_t + \alpha(t)uu_x + \beta(t)u^2u_x + \gamma(t)u_{xxx} = 0, \qquad (t,x) \in \mathbb{R}_+ \times \mathbb{R}, \qquad (1.1)$$

where $\alpha(t),\ \beta(t)\ and\ \gamma(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. $Eq.(1.1)$ arises in many physical phenomena, often simultaneously exist in practical problems such as fluid physics and quantum field theory [5]. If such physical phenomena are considered in random environment, we can get random combined $KdV - mKdV$ equation. In order to give the exact solutions of this random model, we only consider it in white noise environment, that is, we will study the following Wick-type stochastic combined KdV-mKdV equations:

$$U_t + A(t) \diamond U \diamond U_x + B(t) \diamond U^2 \diamond U_x + \Gamma(t) \diamond U_{xxx} = 0, \qquad (1.2)$$

where "$\diamond$" is the Wick product on the Kondratiev distribution space $(s)_{-1}$ and $A(t)$, $B(t)$ and $\Gamma(t)$ are $(s)_{-1}$ -valued functions [24]. It is well known that the solitons are stable against mutual collisions and behave like particles. In this sense, it is very important to study the nonlinear equations in random environment. However, the variable coefficients nonlinear equations, as well as constant coefficients equations, cannot describe the realistic physical phenomena exactly. Wadati [32] first answered the interesting question. " How does external noise affect the motion of solitons?" and studied the diffusion of soliton combined KdV- mKdV equation under Gaussiannoise, which satisfies a diffusion equation in transformed coordinates.

The cauchy problems associated with stochastic partial differential equations (SPDEs) was discussed by many authors, e.g., de Bouard and Debussche [6], Debussche and Printems [8]. Printems [25] and Ghany and Hyder [16]. On the basis of white noise functional analysis [24]. Ghany et al. [13−15, 17−20] studied more intensely the white noise functional solutions for some nonlinear stochastic PDEs. Recently, many new methods have been proposed to solve the nonlinear wave equations such as variational iteration method [9, 10], tanh-function method [11, 33, 37], homotopy perturbation method [12, 26, 34], homotopy analysis method [1], tanh-coth method [27−29] and F-expansion method [2, 31, 35]. The Exp-function method was first proposed in [21]. As it is a straightforward and concise method, it was successfully applied to obtain generalized solitary and periodic wave solutions of some nonlinear evolution equation arising in mathematical physics. The application of this method can be found in [22, 30, 38, 39]. Moreover, The solution procedure of this method, with the aid of *Maple*, is of utter simplicity and this method can be easily extended to other kinds of nonlinear evolution equations [23, 36]. In our paper, we use the Exp-function method to seek new exact traveling wave solutions for the variable coefficients combined KdV-mKdV equation. These solutions include soliton and periodic wave solutions. Then, with the help of Hermite transform and white noise analysis, we employ these solutions to find exact white noise functional solutions for the Wick-type stochastic combined KdV-mKdV equations.

Soliton and Periodic Wave Solutions of Eq.(1.1)

In this section, we apply Hermite transform, white noise theory, and Exp-function method to explore soliton and periodic wave solutions for $Eq.(1.1)$. Applying Hermite transform to $Eq.(1.2)$, we get the deterministic equation:

$$\tilde{U}_t(t,x,z) + \tilde{A}(t,z)\,\tilde{U}(t,x,z)\,\tilde{U}_x(t,x,z) + \tilde{B}(t,z)\tilde{U}^2(t,x,z)\,\tilde{U}_x(t,x,z) + \tilde{\Gamma}(t,z)\,\tilde{U}_{xxx}(t,x,z) = 0 \qquad (2.1)$$

where $z = (z1, z2, \dots) \in (\mathbb{C}^N)$ is a vector parameter. To look for the traveling wave solution of $Eq.(2.1)$, we make the transformations $\tilde{A}(t,z) := A(t,z), \tilde{B}(t,z) := B(t,z), \tilde{\Gamma}(t,z) := \Gamma(t,z)$ and $\tilde{U}(t,x,z) =: u(t,x,z) = u(\xi(t,x,z))$ with $\xi(t,x,z) = kx + \int_0^t \phi(\tau,z)d\tau + c,$ where $k \neq 0$, c are arbitrary constants and $\phi(\tau,z)$ is a nonzero function of the indicated variables to be determined later. Hence, $Eq.(2.1)$ can be transformed into the following ordinary differential equation: $\phi u' + \alpha k u u' + \beta k u^2 u' + \gamma K^3 u''' = 0, \quad (2.2)$

where the prime denote to the differential with respect to ξ. In view of Exp-function method, the solution of $Eq.(2.1)$, can be expressed in the form:

$$u(t,x,z)=\frac{\sum_{n=-c}^{d} a_n \exp(n\xi)}{\sum_{m=-p}^{q} b_n \exp(m\xi)}, \qquad (2.3)$$

where c, d, p and q are positive integers which could be freely chosen and a_n, b_m are unknown constants to be determined later. $Eq.(2.3)$ can be re-written in an alternative form as follows:

$$u(t,x,z) \quad =\frac{a_c \exp(c\xi)+\cdots+a_{-d}\exp(-d\xi)}{b_p \exp(n\xi)+\cdots+b_{-q}\exp(-q\xi)}, \qquad (2.4)$$

To determine the values of c and p, we balance the linear term of highest order of $Eq.(2.2)$ with the highest order nonlinear term. By simple calculation, we have

$$u'''=\frac{c_1 \exp[(c+7p)\xi)]+\cdots}{c_2 \exp[8p\xi]+\cdots}, \qquad (2.5)$$

$$u^2u'=\frac{c_3 \exp[(3c+p)\xi)]+\cdots}{c_4 \exp[4p\xi]+\cdots}, \qquad =\frac{c_3 \exp[(3c+5p)\xi)]+\cdots}{c_4 \exp[8p\xi]+\cdots}, \qquad (2.6)$$

where c_i are determined coefficients only for simplicity. Balancing highest order of exponential function in $Eqs.(2.5)$, (2.6), we have $p=c$. similarly to determine values of d and q, we balance the linear term of lowest order in $Eq.(2.2)$:

$$u'''=\frac{\ldots+d_1 \exp[-(7q+d)\xi)]}{\ldots+d_2 \exp[-8q\xi]}, \qquad (2.7)$$

$$u^2u'=\begin{cases}\dfrac{\ldots+d_3 \exp[-(2q+2d)\xi)]}{\ldots+d_4 \exp[-4q\xi]}\\ \dfrac{\ldots+d_3 \exp[-(5q+3d)\xi)]}{\ldots+d_4 \exp[-8q\xi]}\end{cases} \qquad (2.8)$$

$$= \qquad (2.8)$$

where d_i are determined coefficients only for simplicity. Therefore we can obtain $d=q$. Now, we solve $Eq.(2.1)$ for some particular cases for the constants p, c, d and q.

Case A.

If we set $p=c=1$ and $d=q=1$, then $Eq.(2.4)$ becomes

$$u(t,x,z)=\frac{\begin{bmatrix}a_1 \exp(\xi(t,x,z))+a_0\\ +a_{-1}\exp(-\xi(t,x,z))\end{bmatrix}}{\begin{bmatrix}b_1 \exp(\xi(t,x,z))+b_0\\ +b_{-1}\exp(-\xi(t,x,z))\end{bmatrix}}, \qquad (2.9)$$

In case $b1 \neq 0$ $Eq.(2.5)$ can be simplified as follows equation.

$$u(t,x,z) = \frac{\begin{bmatrix} a1\exp\left(\xi(t,x,z)\right) + a0 \\ + a_{-1}\exp\left(-\xi(t,x,z)\right) \end{bmatrix}}{\begin{bmatrix} b1\exp\left(\xi(t,x,z)\right) + b0 \\ + b_{-1}\exp\left(-\xi(t,x,z)\right) \end{bmatrix}}, \quad (2.10)$$

Substituting $Eq.(2.10)$ into (2.2), we have $\frac{1}{A}[C_3\exp(3\xi) + C_2\exp(2\xi) + C_1\exp(\xi) + C_0 + C_{-1}\exp(-\xi) + C_{-2}\exp(-2\xi) + C_{-3}\exp(-\xi)] = 0$

Where

$A = [\exp(\xi) + b0 + b_{-1}\exp(-\xi)]^4$ $C_3 = -\phi a_0 + \alpha k a_1^2 b_0 + \beta k a_1^3 b_0 + \gamma k^3 a_1 b_0 - \beta k a_1^2 a_0 - \alpha k a_0 a_1 - k^3 a_0 + \phi a_1 b_0$,

$C_2 = -2\beta k a_1 a_0^2 - 2\phi a_{-1} + \alpha k a_1^2 b_0^2 + 2\alpha k a_1^3 b_{-1} + 2\alpha k a_1^2 b_0 a_0 - 8\gamma k^3 a_{-1} + 2\phi a_1 b_{-1} - 4\gamma k^3 a_1 b_0^2 + 2\alpha k a_1^2 b_{-1} - \alpha k a_0^2 - 2\phi b_0 a_0 - 2\alpha k a_1 a_{-1} - 2\beta k a_1^2 a_{-1} + 2\phi a_1 b_0^2 + 4\gamma k^3 b_0 a_0 + 8\gamma k^3 a_1 b_{-1}$,
$C_1 = -5\phi a_{-1} b_0 + \beta k a_1^2 a_{-1} b_0 + 5\beta k a_1^2 a_0 b_{-1} + \phi a_1 b_0^3 - \alpha k a_0^2 b_0 + 6\phi a_1 b_0 b_{-1} - 5\gamma k^3 b_0 a_{-1} + 2\alpha k a_1 b_{-1} a_0 - \beta k a_0^3 - \gamma k^3 a_0 b_0^2 + 3\alpha k a_1^2 b_0 b_{-1} + \alpha k a_1 b_0^2 a_0 + 23\gamma k^3 a_0 b_{-1} - 3\alpha k a_0 a_{-1} - 2\alpha k a_1 a_{-1} b_0 - \phi b_{-1} a_0 + \beta k a_1 a_0^2 b_0 - \phi a_0 b_0^2 - 6\beta k a_1 a_{-1} a_0 - 18\gamma k^3 a_1 b_0 b_{-1} + \gamma k^3 a_1 b_0^3$,
$C_0 = -4\phi a_{-1} b_{-1} + 32\gamma k^3 a_{-1} b_{-1} - 32\gamma k^3 a_1 b_{-1}^2 - 4\alpha k a_{-1} b_0 a_0 - 4\beta k a_0^2 a_{-1} - 2\alpha k a_{-1}^2 - 4\beta k a_1 a_{-1}^2 - 4\phi a_{-1} b_0^2 - 4\gamma k^3 a_{-1} b_0^2 + 4\phi a_1 b_0^2 b_{-1} + 2\alpha k a_1^2 b_{-1}^2 + 4\beta k a_1 b_0^2 b_{-1} + 4\phi a_1 b_{-1}^2 + 4\gamma k^3 a_1 b_0^2 b_{-1} + 4\beta k a_1^2 a_{-1} b_{-1}$, $C_{-1} = -6\phi a_{-1} b_0 b_{-1} - 5\beta k a_0 a_{-1}^2 - \alpha k a_{-1} a_0 b_0^2 + 2\alpha k a_{-1} b_0 b_{-1} a_1 + \beta k a_0^3 b_{-1} - 2\alpha k a_{-1} a_0 b_{-1} - 18\gamma k^3 a_{-1} b_{-1} b_0 - 3\alpha k a_{-1}^2 b_0 - \gamma k^3 a_{-1} b_0^3 + \alpha k a_0^2 b_{-1} b_0 + 5\gamma k^3 b_{-1}^2 a_1 b_0 - \beta k a_{-1}^2 a_1 b_0 - 23\gamma k^3 a_0 b_{-1}^2 + \phi a_0 b_{-1}^2 + \gamma k^3 a_0 b_{-1} b_0^2 - \phi a_{-1} b_0^3 + 5\phi a_1 b_0 b_{-1}^2 - \beta k a_{-1} a_0^2 b_{-1} + 4\phi a_0 b_{-1} b_0^2 + 3\alpha k a_0 a_1 b_{-1}^2 + 6\beta k a_1 a_{-1} a_0 b_{-1}$,
$C_{-2} = 2\phi a_0 b_{-1}^2 b_0 + 2\alpha k a_{-1} b_{-1}^2 a_1 + 4\gamma k^3 a_{-1} b_0^2 b_{-1} - 2\alpha k a_{-1}^2 b_{-1} - 2\phi a_{-1} b_{-1}^2 - 2\phi a_{-1} b_{-1} b_0^2 + 2\phi a_1 b_{-1}^3 - 2\beta k a_{-1}^3 + \alpha k a_0^2 a_1 b_{-1} + 2\beta k a_0^2 a_{-1} b_{-1} - 4\gamma k^3 a_0 b_{-1}^2 b_0 + -8\gamma k^3 a_{-1} b_{-1}^2 + 8\gamma k^3 a_1 b_{-1}^3$.

$$C_{-3} = \gamma k^3 a_0 b_{-1}^3 + \phi a_0 b_{-1}^3 + \alpha k a_0 b_{-1}^2 a_{-1} - \alpha k a_{-1}^2 b_0 b_{-1} - \phi a_{-1} b_0 b_{-1}^2 - \beta k a_{-1}^3 b_0 - \gamma k^3 a_{-1} b_0 b_{-1}^2 + \beta k a_{-1}^2 a_0 b_{-1}, \quad (2.11)$$

Setting the coefficients of $exp(n\xi)$ to be $zero$, we have. $\begin{cases} C_0 = C_1 = C_2 = C_3 = 0, \\ C_{-1} = C_{-2} = C_{-3} = 0 \end{cases}$ · solving the system by *maple* , we obtain two set of solutions.

$$\begin{cases} a_0 = \dfrac{\begin{bmatrix} b_0[2\beta(t,z)a_1^2 + \alpha(t,z) \\ a_1 + 6\gamma(t,z)k^2] \end{bmatrix}}{2\beta(t,z)a_1 + \alpha(t,z)}, \\ b_{-1} = \begin{bmatrix} \dfrac{\begin{matrix} b_0^2\ [6k^2\beta(t,z) + 4\beta^2(t,z)a_1^2 \\ +4a_1\alpha(t,z)\beta(t,z) + \alpha^2(t,z)] \end{matrix}}{\begin{matrix} 4[\beta^2(t,z)a_1^2 + 4a_1\alpha(t,z) \\ \beta(t,z) + \alpha^2(t,z)] \end{matrix}} \end{bmatrix}, \\ a_{-1} = \dfrac{\begin{bmatrix} a_1 b_0^2\ [6k^2\beta(t,z)\gamma(t,z)) + 4\beta^2(t,z) \\ a_1^2 + 4a_1\alpha(t,z)\beta(t,z) + \alpha^2(t,z)] \end{bmatrix}}{\begin{matrix} 4[\beta^2(t,z)a_1^2 + 4a_1\alpha(t,z) \\ \beta(t,z) + \alpha^2(t,z)] \end{matrix}}, \\ \boldsymbol{\phi} = -[\mathrm{k}a_1^2\beta(t,z) + \mathrm{k}\,a_1\,\alpha(t,z) + k^3\gamma(t,z)]. \end{cases} \quad (2.11)$$

(2.12)

And

$$\begin{cases} a_1 = \left[-\dfrac{\alpha(t,z)}{2\beta(t,z)} + k\dfrac{\sqrt{-6\beta(t,z)}}{\beta(t,z)}\right], \\ a_{-1} = -b_{-1}\left[\dfrac{\alpha(t,z)}{2\beta(t,z)} + k\dfrac{\sqrt{-6\beta(t,z)}}{\beta(t,z)}\right], \\ \boldsymbol{\phi} = \dfrac{k\left[\alpha^2(t,z) + 8k^2\beta(t,z)\gamma(t,z))\right]}{4\beta(t,z)}, \\ a_0 = b_0 = 0. \end{cases} \quad (2.13)$$

so we obtain the following solutions:

$$u_1(t,x,z) = \frac{[\Omega_1 + \Omega_2]}{\begin{bmatrix} a_1 \exp(\xi_1(t,x,z)) \\ + b_0 + \dfrac{1}{a_1\gamma(t,z)}\ \Omega_2 \end{bmatrix}}, \quad (2.14)$$

Where

$$\Omega_1 = \frac{\begin{bmatrix} b_0[2\beta(t,z)a_1^2 + \alpha(t,z) \\ a_1 + 6\gamma(t,z)k^2] \end{bmatrix}}{2\beta(t,z)a_1 + \alpha(t,z)} + a_1 \exp(\xi_1),$$

$$\Omega_2 = \frac{\begin{bmatrix} a_1 b_0^2\ [6k^2\beta(t,z)\gamma(t,z)) \\ +4\beta^2(t,z)a_1^2 + 4a_1 \\ \alpha(t,z)\beta(t,z) + \alpha^2(t,z)] \end{bmatrix}}{\begin{bmatrix} 4[\beta^2(t,z)a_1^2 + 4a_1\alpha(t,z) \\ \beta(t,z) + \alpha^2(t,z)] \end{bmatrix}} \exp(-\xi_1),$$

the above equation we can reduced to the following equation.

$$u_1^* = \left[a_1 + \frac{\left[\frac{6k^2 b_0 \gamma(t,z)}{\alpha(t,z) + 2a_1\beta(t,z)}\right]}{\begin{bmatrix}\exp(\xi_1(t,x,z)) \\ + b_0 + \frac{1}{a_1}\ \Omega_2\end{bmatrix}} \right], \qquad (2.15)$$

The second solution in case $b_0 = 0$ we could obtain the following solution.

$$u_2(t,x,z) = \frac{\lfloor \Omega_3 + \Omega_4 \rfloor}{\begin{bmatrix}\exp(\xi_2(t,x,z)) + b_{-1} \\ \exp(-\xi_2(t,x,z))\end{bmatrix}}, (2.16)$$

Where$\Omega_3 = \left[-\frac{\alpha(t,z)}{2\beta(t,z)} + k\frac{\sqrt{-6\beta(t,z)}}{\beta(t,z)}\right]\exp(\xi_2)$, $\Omega_4 = -b_{-1}\left[\frac{\alpha(t,z)}{2\beta(t,z)} + k\frac{\sqrt{-6\beta(t,z)}}{\beta(t,z)}\right]\exp(-\xi_2)$ Where

$$\xi_1(t,x,z) = \mathrm{k}[\mathrm{x} - \Omega_5], \qquad (2.17)$$

$$\Omega_5 = \int_0^t \begin{bmatrix} a_1^2\beta(\tau,z) + a_1\,\alpha(\tau,z) \\ + k^2\gamma(\tau,z)\end{bmatrix} d\tau.$$

$$\xi_2(t,x,z) = \mathrm{k}[\mathrm{x} + \Omega_6], \qquad (2.18)$$

$$\Omega_6 = \frac{1}{4}\left\{\int_0^t \frac{\lfloor \alpha^2(\tau,z) + 8k^2\beta(\tau,z)\gamma(\tau,z))\rfloor}{\beta(\tau,z)}\right\} d\tau,$$

when k is an imaginary number, using the

transform, $k = iK$

$\exp(\xi_{1,2}) = \exp(iK\,\psi_{1,2}) = \cos(K\psi_{1,2}) + i\sin(K\psi_{1,2})$, $\exp(-\xi_{1,2}) = \exp(-iK\,\psi_{1,2})$
$= \cos(K\psi_{1,2}) - i\sin(K\psi_{1,2})$. Then we obtain the periodic solutions as following.

$$u_3(t,x,z) = \left[a_1 + \frac{\left[\frac{6k^2 b_0 \gamma(t,z)}{\alpha(t,z) + 2a_1\beta(t,z)}\right]}{\begin{bmatrix}\Omega_7\cos(K\psi_1) + b_0 \\ +\mathrm{i}\,\Omega_8 \sin(K\psi_1)\end{bmatrix}} \right], \qquad (2.19)$$

Where

$$\Omega_7 = \left[1 + \left[\frac{b_0^2\,[-6K^2\beta(t,z)\gamma(t,z) + [\alpha(t,z) + 2a_1\beta(t,z)]^2]}{4[\alpha(t,z) + 2a_1\beta(t,z)]^2} \right] \right],$$

$$\Omega_8 = \left[1 - \left[\frac{b_0^2\,[-6K^2\beta(t,z)\gamma(t,z) + [\alpha(t,z) + 2a_1\beta(t,z)]^2]}{4[\alpha(t,z) + 2a_1\beta(t,z)]^2}\right]\right],$$

the periodic wave solution. $Eq. (2.19)$, might have some potential applications. For practical use, we eliminate the imaginary part in $Eq. (2.19)$ this requires (i.e)

$$\left[\frac{b_0^2\,[-6K^2\beta(t,z)\gamma(t,z) + [\alpha(t,z) + 2a_1\beta(t,z)]^2]}{4[\alpha(t,z) + 2a_1\beta(t,z)]^2}\right] = 1,$$

$$K^2 = \frac{[b_0^2 - 4][\alpha(t,z) + 2a_1\beta(t,z)]^2}{6b_0^2\,\beta(t,z)\gamma(t,z)},$$

then $Eq. (2.19)$ is reduced to periodic wave solution as following.

$$u_3^*(t,x,z) = a_1 - \frac{[b_0^2 - 4][\alpha(t,z) + 2a_1\beta(t,z)]}{b_0\beta(t,z)[2\cos(K\psi_1) + b_0]}, (2.20)$$

similarly.$Eq. (2.15)$ can be translated in to periodic or compact like solutions as follows.

$$u_4(t,x,z) = \frac{\left[\begin{array}{c}\Omega_9\cos(K\psi_2) + \\ \Omega_{10}\sin(K\psi_2)\end{array}\right]}{\left[\begin{array}{c}[1 + b_{-1}]\cos(K\psi_2) + \\ i[1 - b_{-1}]\sin(K\psi_2)\end{array}\right]}, (2.21)$$

Where

$$\Omega_9 = \left\{-\frac{\alpha(t,z)}{2\beta(t,z)}[1 + b_{-1}] + K\frac{\sqrt{6\beta(t,z)}}{\beta(t,z)}[1 - b_{-1}]\right\},$$

$$\Omega_{10} = -i\left\{-\frac{\alpha(t,z)}{2\beta(t,z)}[1 - b_{-1}] + K\frac{\sqrt{6\beta(t,z)}}{\beta(t,z)}[1 + b_{-1}]\right\},$$

elimination of the imaginary part in present $Eq. (2.19)$ requires setting $[b_{-1} = 1]$ for simplicity, we obtain the following periodic wave solution:

$$u_4^*(t,x,z) = \frac{\begin{bmatrix} -[\frac{\alpha(t,z)}{2\beta(t,z)}\cos(K\psi_2) \\ + K\frac{\sqrt{6\beta(t,z)}}{\beta(t,z)}\sin(K\psi_2)] \end{bmatrix}}{\cos(K\psi_2)}, \qquad (2.22)$$

where

$$\psi_1(t,x,z) = [x - \Omega_{11}], \qquad (2.23)$$

$$\Omega_{11} = \int_0^t [a_1^2\beta(\tau,z) + a_1\,\alpha(\tau,z) - K^2\gamma(\tau,z)]\,d\tau.$$

$$\psi_2(t,x,z) = [x + \Omega_{12}], \qquad (2.24)$$

$$\Omega_{12} = \frac{1}{4}\int_0^t \frac{\lfloor\alpha^2(\tau,z) - 8K^2\beta(\tau,z)\gamma(\tau,z))\rfloor}{\beta(\tau,z)}\,d\tau,$$

Remark that: there are infinitely number of soliton and periodic wave solutions for $Eq.(2.1)$. These solutions come from setting different values for the positive integers $p,\ c,\ d\ and\ q$. The above mentioned cases are just to clarify how far our technique is applicable.

Exact White Noise Functional Solutions of Eq. (1.2)

In this section, we employ the results of Section 2 and Hermite transform to obtain exact white noise functional solutions for Wick-type stochastic combined KdV-mKdV equations(1.2). The properties of exponential and trigonometric functions yield that there exists a bounded open set $\boldsymbol{D} \subset \mathbb{R}_+ \times \mathbb{R}$, $\rho < \infty$, $\lambda > 0$ such that the solution $u(t,x,z)$ of $Eq.(2.1)$ and all its partial derivatives which are involved in $Eq.(2.1)$ are uniformly bounded for $(t,x,z) \in \boldsymbol{D} \times K_\rho(\lambda)$, continuous with respect to $(t,x) \in \boldsymbol{D}$ for all $z \in K_\rho(\lambda)$ and analytic with respect to $z \in K_\rho(\lambda)$, for all $(t,x) \in \boldsymbol{D}$. From Theorem 4.1.1 in [24], there exists $U(t,x,z) \in (S)_{-1}$ such that $u(t,x,z) = \tilde{U}(t,x)(z)$ for all $(t,x,z) \in \boldsymbol{D} \times K_\rho(\lambda)$, and $U(t,x)$ solves $Eq.(1.2)$ in $(S)_{-1}$ Hence, by applying the inverse Hermite transform to the results of Section 2, we get the exact white noise functional solutions of $Eq.(1.2)$ as follows:

Exact Stochastic Soliton wave Solutions:

$$U_1(t,x) = \left[a_1 + \frac{\left[\frac{6k^2 b_0 \Gamma(t)}{A(t) + 2a_1 B(t)}\right]}{\begin{bmatrix}\exp^{\diamond}(\Xi_1(t,x)) \\ + b_0 + \Omega_{13}\end{bmatrix}}\right], \qquad (3.1)$$

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Where

$$\Omega_{13} = \frac{\begin{bmatrix} b_0^2\ [6k^2 B(t)\Gamma(t) \\ +[A(t)+2a_1 B(t)]^{\diamond 2}] \end{bmatrix}}{[4[A(t)+2a_1 B(t)]^{\diamond 2}]]} \diamond \exp^{\diamond}(-\Xi_1(t,x)), \quad U_2(t,x) = \frac{\lfloor \Omega_{14} + \Omega_{15} \rfloor}{\begin{bmatrix} \exp^{\diamond}(\Xi_2(t,x)) + b_{-1} \\ \exp^{\diamond}(-\Xi_2(t,x)) \end{bmatrix}}, \qquad (3.2)$$

Where

$$\Omega_{14} = \left[-\frac{A(t)}{2B(t)} + k\frac{\sqrt{-6B(t)}}{B(t)}\right] \diamond \exp^{\diamond}(\Xi_2(t,x)),$$

$$\Omega_{15} = -b_{-1}\left[\frac{A(t)}{2B(t)} + k\frac{\sqrt{-6B(t)}}{B(t)}\right] \diamond \exp^{\diamond}(-\Xi_2(t,x)),$$

Where

$$\Xi_1(t,x) = \mathrm{k}[\mathrm{x} - \Omega_{16}], \qquad (3.3)$$

$$\Omega_{16} = \int_0^t [a_1^2 B(\tau) + a_1 A(\tau) + k^2 \Gamma(\tau)]\mathrm{d}\tau.$$

$$\Xi_2(t,x) = \mathrm{k}[\mathrm{x} + \Omega_{17}], \qquad (3.4)$$

$$\Omega_{17} = \frac{1}{4}\int_0^t \frac{\lfloor A^{\diamond 2}(\tau) + 8k^2 B(\tau) \diamond \Gamma(\tau)) \rfloor}{B(\tau)},$$

Exact Stochastic Periodic wave Solutions:

$$U_3(t,x) = \left[a_1 + \frac{\left[\frac{-6K^2 b_0 \Gamma(t)}{A(t)+2a_1 B(t)}\right]}{\begin{bmatrix} \Omega_{18} \diamond \cos^{\diamond}(K\Psi_1) + b_0 \\ +i\,\Omega_{19} \diamond \sin^{\diamond}(K\Psi_1) \end{bmatrix}}\right], \qquad (3.5)$$

Where

$$\Omega_{18} = \left[1 + \left[\frac{\begin{matrix} b_0^2\ [-6K^2 B(t) \diamond \Gamma(t) \\ +[A(t)+2a_1 B(t)]^{\diamond 2}] \end{matrix}}{4[A(t)+2a_1 B(t)]^{\diamond 2}}\right]\right], \quad \Omega_{19} = \left[1 - \left[\frac{\begin{matrix} b_0^2\ [-6K^2 B(t) \diamond \Gamma(t) \\ +[A(t)+2a_1 B(t)]^{\diamond 2}] \end{matrix}}{4[A(t)+2a_1 B(t)]^{\diamond 2}}\right]\right].$$

$$U_3^{*}(t,x) = a_1 - \frac{[b_0^2 - 4][A(t)+2a_1 B(t)]}{b_0 B(t) \diamond [2\cos^{\diamond}(K\Psi_1) + b_0]}, \qquad (3.6)$$

$$U_4(t,x,) = \frac{\begin{bmatrix}\Omega_{20} \diamond \cos^{\diamond}(K\Psi_2) + \\ \Omega_{21} \diamond \sin^{\diamond}(K\Psi_2)\end{bmatrix}}{\begin{bmatrix}[1+b_{-1}]\cos^{\diamond}(K\Psi_2) + \\ i[1-b_{-1}]\sin^{\diamond}(K\Psi_2)\end{bmatrix}}, \qquad (3.7)$$

Where

$$\Omega_{20} = \left\{-\frac{A(t)}{2B(t)}[1+b_{-1}] \qquad -K\frac{\sqrt{6B(t)}}{B(t)}[1-b_{-1}]\right\},$$

$$\Omega_{21} = i\left\{\frac{A(t)}{2B(t)}[1-b_{-1}] \qquad +K\frac{\sqrt{6B(t)}}{B(t)}[1+b_{-1}]\right\},$$

$$U_4^{*}(t,x) = \frac{\begin{bmatrix}-[\frac{A(t)}{2B(t)} \diamond \cos^{\diamond}(K\Psi_2) \\ + K\frac{\sqrt{6B(t)}}{B(t)} \diamond \sin^{\diamond}(K\Psi_2)]\end{bmatrix}}{\cos^{\diamond}(K\Psi_2)}, \qquad (3.8)$$

where

$$\Psi_1(t,x) = [x - \Omega_{22}], \qquad (3.9)$$

$$\Omega_{22} = \int_0^t [a_1^2 B(\tau) + a_1 A(\tau) - K^2 \Gamma(\tau)]d\tau.$$

$$\Psi_2(t,x) = [x + \Omega_{23}], \qquad (3.10)$$

$$\Omega_{23} = \frac{1}{4}\int_0^t \frac{[A^{\diamond 2}(\tau) - 8K^2 B(\tau) \diamond \Gamma(\tau))]}{B(\tau)} d\tau,$$

We observe that. For different forms of $A(t), B(t)$ and $\Gamma(t)$, we can get different exact white noise functional solutions of $Eq.\,(1.2)$ from $Eqs.\,(3.1) - (3.10)$.

Example

It is well known that Wick version of function is usually difficult to evaluate. So, in this section, we give non-Wick version of solutions of $Eq.\,(1.2)$. Let $W_t = \dot{B}_t$ be the Gaussian white noise, where B_t is the Brownian motion. We have the Hermite transform $\widetilde{W}_t(z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$ [24]. Since $\exp^{\diamond}(B_t) = \exp\left(B_t - \frac{t^2}{2}\right)$, we have $\sin^{\diamond}(B_t) = \sin\left(B_t - \frac{t^2}{2}\right)$ and $\cos^{\diamond}(B_t) = \cos\left(B_t - \frac{t^2}{2}\right)$.
Suppose $A(t) = \nu_1\Gamma(t),\ B(t) = \nu_2\Gamma(t)$, and $\Gamma(t) = \mu(t) + \nu_3 W_t$ where $\nu_1,\ \nu_2$ *and* ν_3 are arbitrary constants and $\mu(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$.

Therefore, for $A(t)B(t)\Gamma(t) \neq 0$, Exact white noise functional solutions of $Eq.(1.2)$ are as follows:

$$U_5(t,x) = \left[a_1 + \frac{\left[\frac{6k^2 b_0}{\nu_1 + 2a_1\nu_2}\right]}{\left[\begin{matrix}\exp(\Xi_3(t,x))\\ + b_0 + \Omega_{24}\end{matrix}\right]} \right], \qquad (4.1)$$

Where

$$\Omega_{24} = \frac{b_0^2}{4}[1 + \frac{6\nu_2 k^2}{[\nu_1 + 2a_1\nu_2]}]\exp(-\Xi_3(t,x)) \qquad U_6(t,x) = \frac{[\Omega_{25} - \Omega_{26}]}{\left[\begin{matrix}\exp(\Xi_4(t,x)) + \\ b_{-1}\exp(-\Xi_4(t,x))\end{matrix}\right]}, \qquad (4.2)$$

Where

$$\Omega_{25} = \left[-\frac{\nu_1}{2\nu_2} + k\sqrt{[\frac{-6}{\nu_2[\mu(t) + \nu_3 W_t]}]}\right]\exp(\Xi_4(t,x)),$$

$$\Omega_{26} = b_{-1}\left[\frac{\nu_1}{2\nu_2} + k\sqrt{[\frac{-6}{\nu_2[\mu(t) + \nu_3 W_t]}]}\right]\exp(-\Xi_4(t,x)),$$

Where

$$\Xi_3(t,x) = \mathrm{k}[\mathrm{x} - \Omega_{27}], \qquad (4.3)$$

$$\Omega_{27} = [\nu_1 a_1 + \nu_2 {a_1}^2 + k^2]\left\{\int_0^t \mu(t)\mathrm{d\tau} \qquad + \nu_3\left(B_t - \frac{t^2}{2}\right)\right\},$$

$$\Xi_4(t,x) = \mathrm{k}[\mathrm{x} + \Omega_{28}], \qquad (4.4)$$

$$\Omega_{28} = \frac{{\nu_1}^2 + 8k^2\nu_2}{4\nu_2}\left\{\int_0^t \mu(t)\mathrm{d\tau} \qquad + \nu_3\left(B_t - \frac{t^2}{2}\right)\right\},$$

$$U_7(t,x) = \left[a_1 + \frac{\left[\frac{6k^2 b_0}{\nu_1 + 2a_1\nu_2}\right]}{\left[\begin{matrix}\Omega_{29}\cos(K\Psi_3) + b_0\\ + \mathrm{i}\,\Omega_{30}\sin(K\Psi_3)\end{matrix}\right]} \right], \qquad (4.5)$$

Where

$$\Omega_{29} = \left[1 + \frac{b_0^2}{4}[1 - \frac{6k^2\nu_2}{[\nu_1 + 2a_1\nu_2]^2}\right], \Omega_{30} = \left[1 - \frac{b_0^2}{4}[1 - \frac{6k^2\nu_2}{[\nu_1 + 2a_1\nu_2]^2}\right],$$

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$$U_7^*(t,x) = a_1 - \frac{[b_0^2 - 4][\nu_1 + 2a_1\nu_2]}{b_0\nu_2[2\cos(K\Psi_3) + b_0]}, \qquad (4.6)$$

$$U_8(t,x,) = \frac{\begin{bmatrix}\Omega_{31}\cos(K\Psi_4) + \\ \Omega_{32}\sin(K\Psi_4)\end{bmatrix}}{\begin{bmatrix}[1+b_{-1}]\cos(K\Psi_4) + \\ i[1-b_{-1}]\sin(K\Psi_4)\end{bmatrix}}, \qquad (4.7)$$

Where

$$\Omega_{31} = -\left\{\frac{\nu_1}{2\nu_2}[1+b_{-1}] + K\sqrt{[\frac{6}{\nu_2[\mu(t)+\nu_3 W_t]}]}[1-b_{-1}]\right\}, \Omega_{32} = i\left\{\frac{\nu_1}{2\nu_2}[1-b_{-1}] + K\sqrt{[\frac{6}{\nu_2[\mu(t)+\nu_3 W_t]}]}[1+b_{-1}]\right\}. U_8^*(t,x) = \frac{\begin{bmatrix}-[\frac{\nu_1}{2\nu_2}\cos(K\Psi_4) + K \\ \sqrt{\frac{6}{\nu_2[\mu(t)+\nu_3 W_t]}}\sin(K\Psi_4)]\end{bmatrix}}{\cos(K\Psi_4)}, \qquad (4.8)$$

Where

$$\Psi_3(t,x) = [x - \Omega_{33}], \qquad (4.9)$$

$$\Omega_{33} = [\nu_1 a_1 + \nu_2 {a_1}^2 - K^2]\left\{\int_0^t \mu(t)d\tau + \nu_3\left(B_t - \frac{t^2}{2}\right)\right\},$$

$$\Psi_4(t,x) = [x + \Omega_{34}], \qquad (4.10)$$

$$\Omega_{34} = \frac{{\nu_1}^2 - 8K^2\nu_2}{4\nu_2}\left\{\int_0^t \mu(t)d\tau + \nu_3\left(B_t - \frac{t^2}{2}\right)\right\},$$

Conclusion

This paper is devoted to implement new strategies. That gives exact white noise functional solutions for the variable coefficients Wick-type stochastic combined KdV-mKdV equations. The strategies pursued in this work rest mainly on Hermite transform, white noise analysis and Exp-function method, all of which are employed to find exact white noise functional solutions of $Eq. (1.2)$. Moreover, the planner which we have proposed in this paper can be also applied to other nonlinear (PDEs) in mathematical physics, for example. KdV, mKdV, KdVB, sKdV, Sawada-Kotera, Zhiber-Shabat, Zakharov-Kuznetsov and BBM equations, we can obtain a new set of exact white noise functional solutions for the Wick-type stochastic combined KdV-mKdV equations by using another methods in $Eq. (1.1)$. Note that, there is a unitary mapping between the Gaussian white noise space and the Poisson white noise space, this connection was given by Benth and Gjerde [7] . Hence, with the help of this connection, we can derive some Poisson white noise functional solutions, if the coefficients $A(t)$, $B(t)$ and $\Gamma(t)$ are Poisson white noise functions in $Eq. (1.2)$.

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